1. A manager in a sweet factory believes that the machines are working incorrectly and the proportion p of underweight bags of sweets is more than 5%. He decides to test this by randomly selecting a sample of 5 bags and recording the number x that are underweight. The manager sets up the hypotheses H_0: $p = 0.05$ and H_1: $p > 0.05$ and rejects the null hypothesis if $x > 1$.

(a) Find the size of the test.

(2)

(b) Show that the power function of the test is

$$1 - (1 - p)^4(1 + 4p)$$

(3)

The manager goes on holiday and his deputy checks the production by randomly selecting a sample of 10 bags of sweets. He rejects the hypothesis that $p = 0.05$ if more than 2 underweight bags are found in the sample.

(c) Find the probability of a Type I error using the deputy's test.

(2)

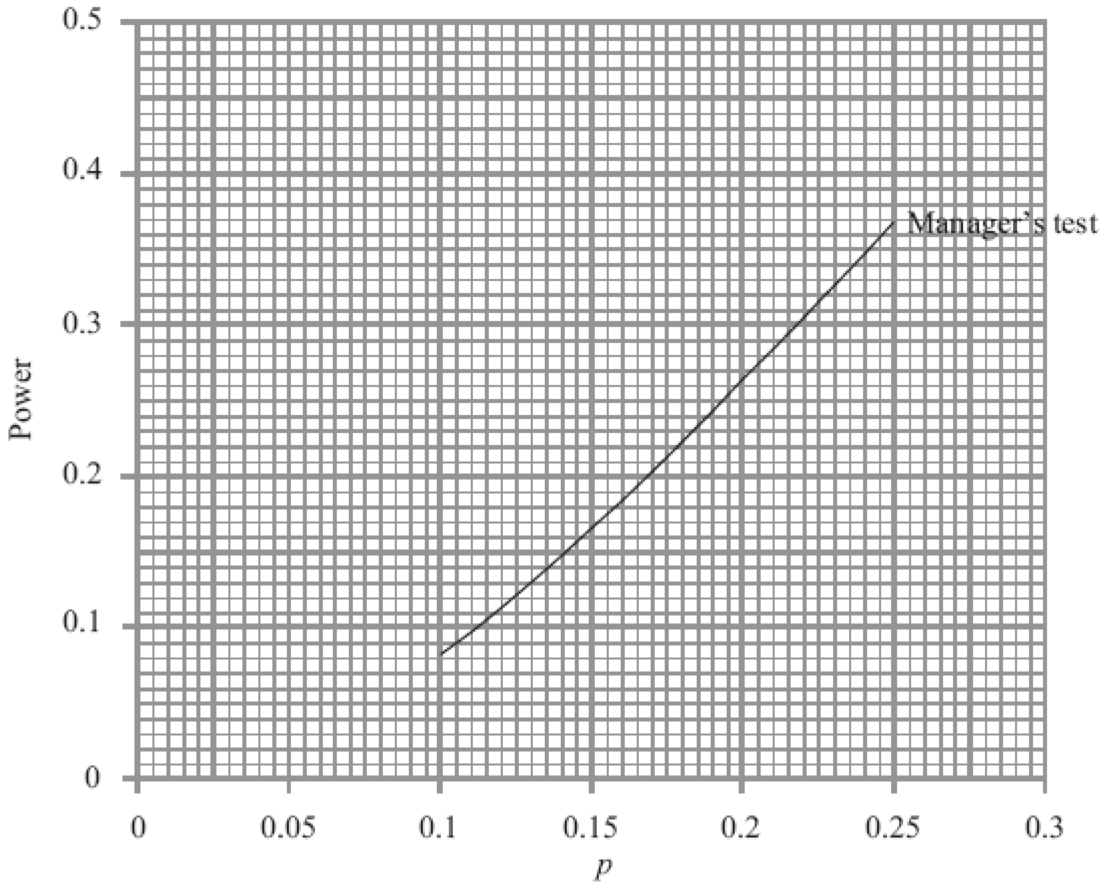
The table below gives some values, to 2 decimal places, of the power function for the deputy's test.

p	0.10	0.15	0.20	0.25
Power	0.07	s	0.32	0.47

(d) Find the value of s.

(1)

The graph of the power function for the manager's test is shown the diagram below.



(e) On the same axes, draw the graph of the power function for the deputy's test.

(1)

(f) (i) State the value of p where these graphs intersect.

(ii) Compare the effectiveness of the two tests if p is greater than this value.

(2)

The deputy suggests that they should use his sampling method rather than the manager's.

(g) Give a reason why the manager might not agree to this change.

(1)
(Total 12 marks)

2. A farmer set up a trial to assess the effect of two different diets on the increase in the weight of his lambs. He randomly selected 20 lambs. Ten of the lambs were given diet A and the other 10 lambs were given diet B. The gain in weight, in kg, of each lamb over the period of the trial was recorded.

(a) State why a paired t-test is not suitable for use with these data.

(1)

(b) Suggest an alternative method for selecting the sample which would make the use of a paired t-test valid.

(1)

(c) Suggest two other factors that the farmer might consider when selecting the sample.

(2)

The following paired data were collected.

Diet A	5	6	7	4.6	6.1	5.7	6.2	7.4	5	3
Diet B	7	7.2	8	6.4	5.1	7.9	8.2	6.2	6.1	5.8

(d) Using a paired t-test, at the 5% significance level, test whether or not there is evidence of a difference in the weight gained by the lambs using diet A compared with those using diet B.

(8)

(e) State, giving a reason, which diet you would recommend the farmer to use for his lambs.

(1)
(Total 13 marks)

1.	(a)	$X \sim B(5, p)$		
		Size = P(reject H_0/ $p = 0.05$)		
		$= P(X > 1/ p = 0.05)$		
		$= 1 - 0.9774$	M1	
		$= 0.0226$	A1	2

Note

M1 for finding P (X>1)
A1 awrt 0.0226

M1 for finding P(Y > 2)
A1 awrt0.0115

	(b)	Power = $1 - P(0) - P(1)$	M1	
		$= 1 - (1 - p)^5 - 5(1 - p)^4 p$	M1	
		$= 1 - (1 - p)^4 (1 - p + 5p)$		
		$= 1 - (1 - p)^4 (1 + 4p)$	A1cso	3

Note

M1 for 1-P(0) – P(1)
M1 for $1 - (1 - p)^5 - 5(1 - p)^4 p$
A1 cso

B1 0.18 cao

	(c)	$Y \sim B(10, p)$		
		P (Type I error) = $P(Y > 2/p = 0.05)$	M1	
		$= 1 - 0.9885$		
		$= 0.0115$	A1	2

Note

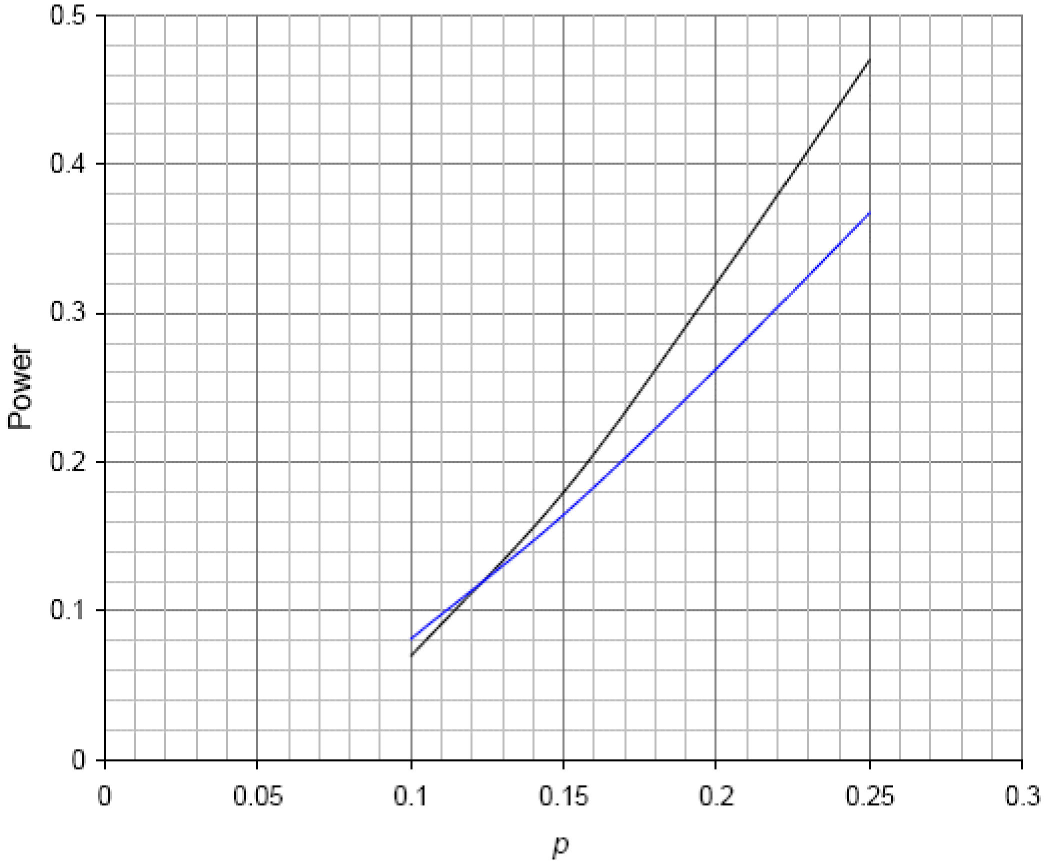
B1 graph. ft their value of s

	(d)	$s = 0.18$	B1	1

Note

B1 ft their intersection.
B1 deputy test more powerful o.e.

(e)



	B1ft	1

Note

If give first statement they must suggest p unlikely to be above 0.12

(f)	(i)	intersection 0.12 – 0.13 "their graphs intersection"	B1ft	
	(ii)	if $p > 0.12$ the deputy's test is more powerful.	B1	2
(g)		More powerful for $p < 0.12$ and p unlikely to be above 0.12 Allow it would cost more/take longer/more to sample	B1	1

[12]

2.	(a)	The data use not collected in pairs	B1	1
	(b)	Use data from twin lambs	B1	1
	(c)	Age, weight, gender *Any two sensible factors*	B1; B1	2

(d)	$d = B - A$		
	d = 2, 1.2, 1, 1.8, –1, 2.2, 2, –1.2, 1.1, 2.8	M1	
	$\sum d = 11.9$; $\sum d^2 = 30.01$		
	$\therefore \bar{d} = 1.19$; $s^2 = 1.761$ (s 1.327)	A1; A1	
	H_0: $\delta = 0$; H_1: $\delta \neq 0$	both B1	
	Allow μ_D for δ		
	$T = \dfrac{1.19 - 0}{\sqrt{1.761/10}} = 2.83574\ldots$	M1	
		A1	
	Awrt 2.84 for A1		
	$\propto = 9$; CV: $t = 2.262$	B1	
	Since 2.8357… is in the critical region ($t > 2.262$) there is evidence to reject H_0. The (mean) weight gained by the lambs is different for each diet.	A1ft	8
	Using non-paired t-test.		
	H_0: $\mu_A = \mu_B$; H_1: $\mu_A \neq \mu_B$	B1	
	$t = \dfrac{\mu_A - \mu_B}{\sqrt{s_p^2\left(\frac{1}{10} + \frac{1}{10}\right)}} = -2.30$	B1	
	Awrt –2.30		
	CV: $\lvert t \rvert = 2.101$	B1	
	Conclusion: Mean weight gained is different	B1	4
	NB $\mu_A = 5.16$ $\mu_B = 6.79$ $s_p^2 = 1.342722..$		
(e)	Diet B	B1	1

[13]

1. Many candidates were able to gain full marks in this question and even those who were unable to answer parts (a) to (c) gained several marks in the latter parts.

In part (b) a complete solution was often seen although several candidates wrote Power = 1 – P(0) – P(1) and then concluded that Power = $1 - (1 - p)^4(1 + 4p)$ with no steps in between. This did not gain full marks.

In part (d) several candidates used the power function given in part (b) rather than find the power for the deputy's test using the tables.

2. The first three parts of this question were not well answered with many not knowing the conditions for the use of a paired *t*-test. Many could not relate to the practical aspects of this question. Apart from poor arithmetic part (d) was usually correct although the conclusion was not always in context. The correct diet was usually stated in part (e) but the reason was not always convincing.